for all x. Then (2) and (3) yield the result that $V \leqq V(0)e^{-kt}$, whence $V(x) \rightarrow 0$ as $t \rightarrow \infty$. If $V(x)$ is a function such as $\sum_i x_i^2$, with the property that $V(x) \rightarrow 0$ if and only if $x \rightarrow 0$, we have deduced in this way the important fact that $x(t) \rightarrow 0$ as $t \rightarrow \infty$, a stability result.

The problem, of course, lies in obtaining $V(x)$, given $g(x)$. Although there is no uniform approach, there exists a vast literature of results due to mathematicians such as Cetaev, Malkin, Persidskii, Massera, Letov, and others. An excellent survey may be found in another recent book in this area, namely, J. P. LaSalle and S. Lefschetz, *Stability by Liapunov's Direct Method with Applications*, Academic Press Inc., New York, 1962.

The great merit of Krasovskii's book is to contain not only a more complete and detailed account of the research of this nature in the field of ordinary differential equations, but also to present a thorough discussion of the application of these methods to differential-difference and more general time-lag equations.

The book is wholeheartedly recommended to all those interested in the modern theory of differential equations and in modern control theory.

The format is attractive, the price is reasonable, and the translation by J. L. Brenner is excellent.

RICHARD BELLMAN

90[X].—JAMES B. SCARBOROUGH, *Numerical Mathematical Analysis*, Fifth Edition, Johns Hopkins Press, Baltimore, Md., 1962, xxi + 594 p., 23.5 cm. Price $7.00.

This is a revised edition of the well-known text by James B. Scarborough. In addition to a number of corrections and minor changes, the Fifth Edition contains a chapter on Newton's interpolation formula for unequal intervals. It is gratifying that the author has been able to find time periodically to review and improve one of the oldest and most popular elementary texts in the field of Numerical Analysis.

H. P.

91[X, Z].—GEORGE S. SEBESTYEN, *Decision-Making Processes in Pattern Recognition*, The Macmillan Company, New York, 1962, viii + 162 p., 24 cm. Price $7.50.

Pattern recognition is a subject which is currently receiving considerable attention. It is important in a variety of situations ranging from the need of the Post Office for mechanical reading devices to speed up sorting of the mails to the need of the Military to be able to decide whether an incoming radar or sonar signal comes from a harmless object such as a meteor or a fishing boat, or whether it comes from a threatening source such as a missile warhead or a hostile submarine. In any situation, the problem to be solved is how to organize one's knowledge about the object in question and how to be able to compare this with similarly organized knowledge about the possible categories to which the object can be assigned.

In the book under review, the author attempts to exploit a geometrical point of view. Data describing a given object consist of numerical values assigned to N

parameters. These are regarded as coordinates of a point in N-dimensional Euclidean space. Data from a set of such objects, all members of some class, constitute a cluster of points in N-space. This cluster is characterized by a single parameter: the mean-square distance between all possible independent pairs of points in the cluster. The question whether a new sample data point belongs to the class in question is decided on the basis of its mean-square distance from all points of the cluster.

Pursuing this theme, the author considers first linear, then nonlinear, transformations of the N-dimensional parameter space so as to achieve such intuitively clear and plausible goals as bringing as close together as possible the points of a cluster representing a single class or separating as far as possible the clusters representing two or more classes. Further, he brings in likelihood ratios and some elementary decision theory in his discussion of the problem of setting up criteria for determining whether a given sample data point does or does not belong to a particular class (cluster) or to which of several classes it should be assigned.

Among additional topics taken up is the application of the methods developed here to "learning machines," which could be implemented either as special hardware or as computer programs. This naturally leads to a brief discussion of such closely related topics as neural nets and Perceptrons. There is also a very informative and well illustrated section on the geometric interpretation of various classification procedures.

As a primary example of the application of his methods, the author presents the results of what must have been extensive experimental work on speaker recognition by means of analysis of the audio frequencies involved. Sample problems: to decide which of two speakers uttered a given phrase; to classify a spoken sound as voiced or unvoiced; to distinguish among spoken numerals, whether spoken by male or female voices. The results of these experiments appear to be quite encouraging. The similarity of these speaker recognition problems to the militarily important recognition problems mentioned in the first paragraph of this review need hardly be belabored.

The book as a whole is a well written account of important research in a fascinating field which touches several disciplines: electrical engineering (which is the author's specialty), statistical decision theory, learning devices, and linear algebra and its geometrical interpretations. In the non-mathematical portions the writing is very clear, leaving no doubt as to what the author is trying to say.

In the mathematical portions (Chapters 2 and 3), where the fundamental ideas are developed, the situation is, unfortunately, otherwise. Here the standards of presentation and the logical orderliness of developments are much poorer than in the rest of the book. To the careful reader it becomes obvious that the author lacked sufficient familiarity and facility with linear algebra and matrix theory to handle competently the problems with which he dealt. For example, he failed to make use of a simple theorem of which he actually seems to have been aware, which would have shown up his class separation criteria as being rather poorly formulated. This theorem asserts that the mean-square distance between all pairs of points of a finite set is twice the mean-square distance of all points from the centroid, that is, twice the variance of the set. Furthermore, if, in forming the mean-square distance between all pairs of points, he had counted pairs whose members coincided as well as pairs of independent points, the equations resulting from almost all of his class membership or class separation criteria would have simplified enormously and been capable of *simple explicit solution.* As it is, they are hanging on the brink of multiple degeneracy and, presumably, numerical instability. This applies particularly to the transformation for improving the separation of two classes (pages 40–42), which is left to be determined by the numerical solution of the eigenvector problem for a matrix pair.

These same Chapters 2 and 3 are, furthermore, sprinkled with a number of false statements, whose nature indicates that the author simply had not assimilated all he had crammed from some algebra text. In at least one place (middle of page 46) he finds himself in a bind and resorts to sheer bluff.

Unfortunately, there is not room enough in a review such as this to present all the evidence to support the preceding comments. To do so properly would involve rewriting the two chapters in question. This really should have been done, with competent coaching, of course, before the book was accepted for publication. To the prospective user of this book the reviewer's advice is, "Caveat emptor!"

On the conceptual side, there is a point which should be brought out: in setting up his class membership and class separation criteria, the author has made use of only a small part of the information about a set of points which is contained in the covariance matrix associated with the set, namely, the trace of this matrix, which is the variance of the set. There are N-1 other parameters, the remaining coefficients of the characteristic equation, which also have geometrical interpretations as mean-square areas of triangles formed by three points, mean-square volumes of tetrahedra formed by four points, and so on. Alternatively, there are N eigenvalues whose sum is the variance of the set. Certain well-known symmetric functions of these are again the coefficients of the characteristic equation. Surely, the information contained in these other parameters ought to be usable in building sharper criteria for determining class membership.

An interested reader will probably find, as the reviewer did, that studying this book is a rewarding experience. Perhaps its greatest contribution lies in the stimulation of further research.

R. P. Eddy

Applied Mathematics Laboratory
David Taylor Model Basin
Washington 7, D. C.

92[Z].—Thomas C. Bartee, Irwin L. Lebow, & Irving S. Reed, *Theory and Design of Digital Machines*, McGraw-Hill Book Company, Inc., New York, 1962, ix + 324 p., 23 cm. Price $11.50.

This is a senior or first-year graduate level introductory text on the logic design of digital machines. It does not cover electronic circuit design, components, programming, or arithmetic algorithms, and it discusses numerical representations only briefly in an appendix.

For the purposes of this book, a digital machine is viewed as a system of registers that store binary scalars or vectors and associated combinational switching circuits that produce binary scalar or vector functions of the contents of registers. The basic process is the transfer of the contents or a function of a register into another